Maximum Work Extraction from Solar Tower

Tawit Chitsomboon
School of Mechanical Engineering, Institute of Engineering, Suranaree University of Technology
Muang District, Nakorn Ratchasima 30000, Thailand
Tel: 044-224410, Fax: 044-224413, E-mail: tabon@ccs.sut.ac.th

Abstract

A theoretical model is proposed for predicting the performance and the characteristic of solar tower with regard to its ability to extract power from the flow within. The model is simple to use because it contains only one parameter, namely, the mass flow rate. The no-load prediction compares very well with the full Euler numerical results. When loads are presented the model predicts higher work extraction in the low flow rate region.

1. Introduction

Solar tower (solar chimney) has attained increasing attention worldwide as a means for generating electricity from the sun. The device consists of a tall tower surrounded by a transparent roof and a turbine located near the base of the circular tower. Solar radiation penetrates through the roof and heat the air underneath, causing it to rise through the tall tower (see Fig. 1).

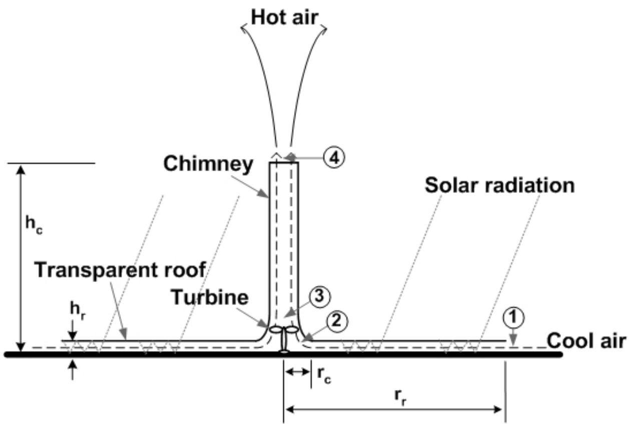

Figure 1: Schematic of Solar Tower

The energy content of the rising hot air can be extracted into shaft power. The mass flow rate (thus velocity) of the air is proportional to the tower height and the amount of absorbed heat as well as other parameters [Refs. 1-3]. Kinetic energy flux of a solar tower system without turbine had been predicted [4] theoretically which compared well with a numerical prediction using Euler's equations as basis. Numerical prediction of turbine work seemed to encounter a maximum limit of the same order as the well known Betz limit of wind turbine technology [5].

Prediction of work extraction with seasonal variation was investigated in [6], using quite detailed thermo-fluid models. The modeled equations are coupled set of algebraic and differential equations which were solved by a numerical procedure. The numerical solution obtained therein, while useful in itself, is not convenient for a parametric study. This paper will attempt to give a much more simplified, but more direct, mathematical model to predict the performance and the maximum work that can be extracted from the turbine of a solar tower. All the important parameters are contained within one model equation so that some qualitative insight for design purpose can perhaps be gained by examining the model.

2. Work Extraction Model

The flow through a solar tower is in the very low Mach number regime; as such, the kinetic energy difference can be safely ignored. Except for the density difference in the vertical part of the chimney which provides the driving force for the system, the density can be assumed to be constant without significant error. With the station numbering as given in Fig. 1 and the mentioned assumptions, the rate of work (power) extracted by the turbine can be accurately determined from the energy equation and the Gibb's relation from classical thermodynamic which can be written as,

$$W^{\bullet} = m\int_3^2 vdP \approx \frac{m}{\rho_3}\int_3^2 dP = \frac{m}{\rho_3}(P_2 - P_3) \quad (1)$$

Therefore, the power goes up with increasing m, P_2 and decreasing P_3. These, however, are contradicting conditions since if P_3 is decreased m will also decreases due to a decrease in the driving force of the flow through the tower.

P_2 , on the other hand, can be managed to be as high as possible subject to the thermo-fluid requirements of the flow.

The relation for P_2 can be obtained by considering the equations for the conservation of mass, momentum and energy of the flow under the roof,

Mass: $\frac{d\rho}{\rho} = -\frac{dV}{V} - \frac{dA}{A}$

Momentum: $\frac{dP}{\rho} = -VdV$

Energy: $\frac{dT}{T} = \frac{\delta Q}{C_p T} - \frac{VdV}{C_p T}$

State: $d\rho/\rho = \frac{dP}{P} - \frac{dT}{T}$

By synthesizing the above governing equations and integrating the differential control volume across the roof length, P_2 can be obtained as,

$$P_2 = P_1 + \int_1^2 dP = P_1 + \int_1^2 \frac{\rho V^2}{1-M^2}\left(\frac{dA}{A} - \frac{\delta Q}{C_p T}\right) \quad (2)$$

where $M^2 = V^2/\gamma RT$. By recognizing that $\delta Q = Q'' dA_r/m$, $P_1 \approx P_\infty$ and that $V = m/\rho A$ the above eq. is re-written as,

$$P_2 = P_\infty + \int_1^2 \frac{m^2}{1-M^2}\frac{dA}{\rho A^3} - \int_1^2 \frac{1}{1-M^2}\frac{m}{\rho}\frac{Q'' dA_r}{C_p T A^2} \quad (3)$$

Q'' , C_p and m are constant while ρ and T can be approximated to be ρ_∞, T_∞ without significantly affecting the numerical values of the terms. The above assumption implies that the irradiation is totally absorbed by the air. While this assumption is not accurate, it does not disqualify the model because a lower irradiation can always be specified to compensate for the various heat loss. A model for heat loss can be devised but it is beyond the scope of this study. The Mach number is assumed to be very low and thus is neglected, the equation is then simplified to be,

$$P_2 = P_1 + \frac{m^2}{\rho_\infty}\int_1^2 \frac{dA}{A^3} - \frac{mQ''}{\rho_\infty C_p T_\infty}\int_1^2 \frac{dA_r}{A^2} \quad (4)$$

If m is known (or assumed) then P_2 can be attained because the integrals can be evaluated from the given geometry of the roof. If dA is negative (it normally is), then P_2 decreases with increasing m and Q'' , as well as geometry of the flow area in relation to the roof area.

A relation for P_3 can be arranged from the momentum equation of the flow in the tower between points 3-4,

$$P_3 - P_4 - \rho_3 gh = \frac{m}{A_3}(V_4 - V_3) + f\frac{h}{d}\frac{\rho_3 V_3^2}{2} \quad (5)$$

Eq. 5 was derived based on a constant-area tower ($A_3 = A_4$). The Hydrostatic equilibrium requires that, $P_4 = P_\infty - \rho_\infty gh$, so the above equation can be written as,

$$P_3 - P_\infty + (\rho_\infty - \rho_3)gh = \frac{mV_3}{A_3}\left(\frac{V_4}{V_3} - 1\right) + f\frac{h}{d}\frac{\rho_3 V_3^2}{2} \quad (6)$$

Even if the tower is a constant area duct the velocity will not be constant along the duct due to change in density with pressure change in an adiabatic environment; this change is not significant so that V_3 alone is used in the Darcy-Weisbach's loss term. The term $(V_4 - V_3)$ is not neglected, however, because an order of magnitude analysis has shown that it is of the same order as the loss term. The term V_4/V_3 can be eliminated in terms of pressure ratio by using the continuity equation and the isentropic relation between 3-4 as,

$$\frac{V_4}{V_3} = \frac{\rho_3}{\rho_4} = \left(\frac{P_3}{P_4}\right)^{1/\gamma} \quad (7)$$

also, $$\rho_3 = \rho_2 \left(\frac{P_3}{P_2} \right)^{1/\gamma} \qquad (8)$$

Together with the mass rate relation $(m = \rho_3 A_3 V_3)$, eq. 6 can be rearranged as,

$$P_3 - P_\infty + [\rho_\infty - \left(P_3 / P_2 \right)^{1/\gamma} \rho_2]gh = \frac{m^2}{\rho_3 A_3{}^2}[\left(P_3 / P_4 \right)^{1/\gamma} - 1] + 0.5 f \frac{h}{d} \frac{m^2}{\rho_3 A_3{}^2} \qquad (9)$$

Knowing (or assuming) m the value of P_3 can be obtained from eq. 9, if P_2 and ρ_2 are known. P_2 is already obtained from eq. 4 and ρ_2 can be computed from the ideal gas equation of state,

$$\rho_2 = P_2 / RT_2 \qquad (10)$$

which needs the value of T_2. The value of T_2 can be estimated from the energy equation across the roof portion,

$$mC_p (T_2 - T_1) + m(\frac{V_2^2}{2} - \frac{V_1^2}{2}) = Q$$

where, for simplicity, frictional effect is ignored because the velocity in this region is quite low. Because the flow is in the very low Mach number regime, the kinetic energy contribution can be safely neglected, therefore,

$$T_2 \approx T_1 + \frac{Q}{mC_p} \qquad (11)$$

which shows that T_2 is also a sole function of m. Putting eqs. 10 and 11 in to eq.9 resulting into an implicit equation for P_3 as a function of m alone. Eqs. 4 and 9 can now be put into eq.1 and computed for the power by specifying the various values of m.

The case without power extraction is also of interest here, since it represents the benchmark case where maximum mass flow rate occurs. The solution for this no-load case can be determined by equating P_2 in eq. 4 to P_3 in eq. 9. This will gives one equation with one unknown (m), albeit implicit one,

$$(P_\infty + \frac{m^2}{\rho_\infty} \int_1^2 \frac{dA}{A^3} - \frac{mQ''}{\rho_\infty C_p T_\infty} \int_1^2 \frac{dA_r}{A^2})(1 - \frac{gh}{RT_2}) - (P_\infty - \rho_\infty gh) = \frac{m^2}{\rho_3 A_3{}^2}[\frac{(P_1 + \frac{m^2}{\rho_\infty} \int_1^2 \frac{dA}{A^3} - \frac{mQ''}{\rho_\infty C_p T_\infty} \int_1^2 \frac{dA_r}{A^2})^{1/\gamma}}{P_4^{1/\gamma}} - 1] + 0.5 f \frac{h}{d} \frac{m^2}{\rho_3 A_3{}^2} \qquad (12)$$

where T_2 is computed from eq. 11. When all integrals are evaluated eq. 12 becomes an algebraic equation in m which can be handily solved for m by trial and error or by a numerical scheme such as the Newton-Raphson's scheme.

3. Results and Discussion

The results of all the test cases presented herein are computed with a level roof assumption (no inclination). Frictional effect is ignored at this stage since other study [7] has shown that the effect of friction is quite low. The baseline case uses the following parameters:

$$Q'' = 550 w/m^2, h_c = 200m,$$
$$R_{roof} = 100m, h_{roof} = 2m.$$

The results of the no-load cases will be presented first. Fig. 2 shows the distributions of the velocities in the tower versus the tower heights at two irradiations, 500 w/m^2 and 800 w/m^2, in order to make a direct comparison with the previous study.

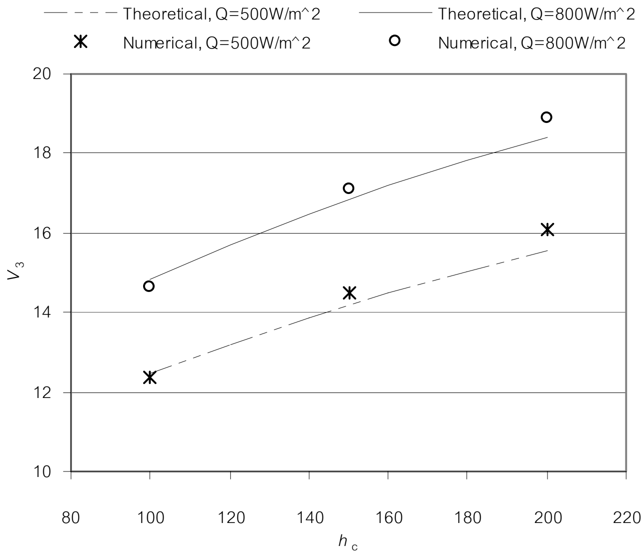

Figure 2: Velocity in Tower Versus Tower height

The numerical results from [4] are used to compare with the theoretical predictions of the present study. The good agreements between the two approaches justify the present theoretical model since the numerical results were obtained from the use of a full Euler's equations.

Another no-load case results are portrayed in Fig. 3 wherein the mass flow rates and the velocities are plotted against the height of the roof at inlet for three irradiations.

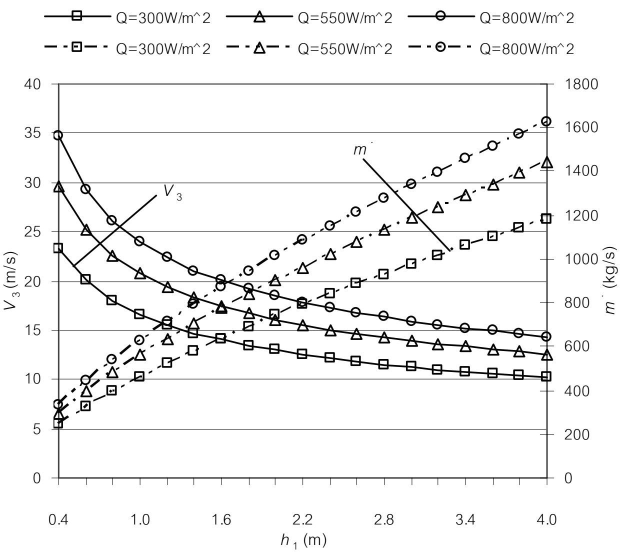

Figure 3: Velocity in Tower Versus Roof Height

It can be seen that velocities decrease with increasing roof height but mass flow rates shows the opposite trends. The value of mass flow rate is important because it directly affects the power output as dictated by eq. 1. It should be noted here that in all the cases computed the cross sectional areas of the towers were set to be equal to the flow area under the roof at the base of the tower. Therefore, a higher roof yields a larger tower.

For the case where there is a work extraction, efficiency is plotted versus the ratio of mass rate between the case in question (m) and the no-load case (m_{nl}). Two kinds of efficiency are shown; one is based on the irradiation and the other on the kinetic power of the no-load case. Fig. 4 depicts the two efficiencies versus the mass rate ratio using roof height as parameter. The irradiation is 550 w/m^2.

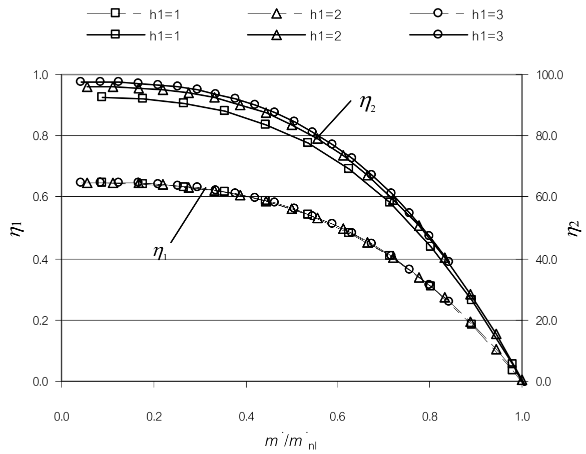

Figure 4: Efficiency Variation with Roof Height

It is evident that the efficiency asymptotes toward the lower end of the mass ratio where maximum work extraction occurs.. Efficiency (of the second kind) does not exhibit a maximum at the Betz limit as numerically forecast in [5] but appears to reach the 100% limit as mass flow rate diminishes. It could be argued that the Betz limit is computed on the basis of the velocity-stage process of a wind turbine in an open environment whereas here the power in a solar tower is extracted on the basis of a pressure-stage process in a closed duct environment. The relationship of the maximum power extraction between the two processes, or the lacking thereof, should be a worthwhile research pursuit. The three data sets from the three roof heights seem to be similar when plotted in this manner. It may not be feasible in real practice to operate a turbine at a very low mass flow rate because the turbine would turn at a very low speed. On the other hand the pressure drop across the turbine would accordingly be relative large, suggesting a unique blade design methodology.

Fig. 5 illustrates efficiencies subject to the variation in irradiations while holding roof height constant at 2 m. Similarity of data sets still remains and are of the same level as in Fig. 4.

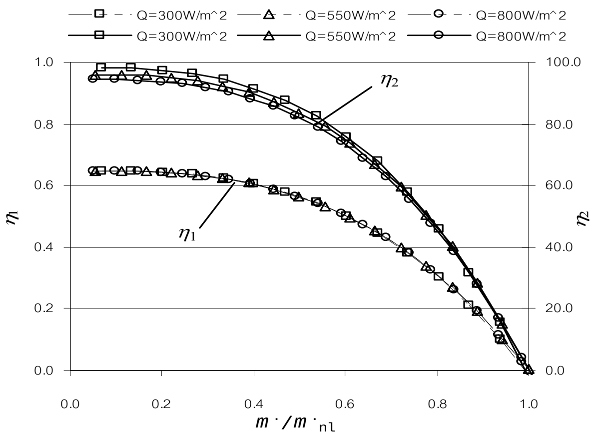

Figure 5: Efficiency Variation with Irradiation

4. Conclusion

For the no-load case, the results from the proposed theoretical model compare very well with the numerical results using full Euler's equations, suggesting the validity of the present model. For the cases with energy extraction, the model predicts higher efficiency in the low mass flow rate region. The model does not predict the Betz limitation on the amount of maximum energy extractable from a solar tower. The model should be useful for parametric study in a preliminary design of a solar tower system.

Acknowledgements

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Nomenclature

A =Cross sectional area

A_r =Roof area

C_p =Specific heat at constant pressure

d =Tower diameter

f =Darcy's friction factor

h_c =Tower height

h_r =Roof height

M =Mach number

m =Mass flow rate

P =Pressure

Q =Total solar flux through roof

Q'' =Irradiation (w/m^2)

q =Heat per unit mass

R =Gas constant

T =Temperature

V =Velocity

$W^{\bullet}$ = Power (work rate)

Greek Symbols

γ =Specific heat ratio

ρ =Density

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